EE360 Project Update, Caleb Kemere, Spring 2001

Problem

It has been shown that OFDM approaches Shannon capacity in frequency-selective fading wireless broadcast channels ((1)). Furthermore, adaptive modulation, power control, and user allocation have proved to result in significant system performance gains in both broadcast and multiple-access situations ((2),(3)). Several questions remain regarding this last result, however. In addition to answering them, this project attempts to analyze the effect of non-linearity (in the form of soft-envelope power limiting) on the capacity of a multiuser OFDM system.

Unanswered Questions

Previous results ((2), (3)) assumed that the channels observed by the users, though random, were uncorrelated between symbols. In a real system, the channel is constantly changing with time. Clearly, an assumption that is typical for these types of analyses, and critical for this paper is that the channel is constant within a single symbol time. However, even this constraint may not be sufficient for a practical system. More likely, in order for the transmission of channel assignment data to not result in significant overhead, these assignments must be constant over several (tens to hundreds, perhaps) symbols. Thus, it is interesting to investigate the performance of the adaptive algorithms in a channel which is correlated in time over several symbols. While it is shown to be a somewhat trivial extension, for similar reasons, the increase in capacity from adaptation in both frequency and time is also observed.

As described above, packet size imposes constraints on the rate that the channel can change, and packet size can be constrained by channel assignment overhead. This suggests that potential system gains may be realized from limiting the amount of unique information necessary for channel assignments. Clearly, each user needs to know whether or not they should try to decode a particular subcarrier in the symbols of a packet. However, if the number of users active within any given packet is limited, one could conceive of an addressing scheme that all users could easily decode that would specify the (small) number of users, and then their particular subcarrier assignments. Thus, while it is clearly beneficial to have users optimally share time-slots in a TDMA OFDM system, it is open question as to what extent multiuser-diversity is required for increased performance.

Clipping Capacity

It appears that little study has been made of the impact of optimal adaptation on the classical problems of OFDM. Some examples of these include: symbol synchronization, oscillator phase noise, and peak-to-average ratio (PAR). In this portion of the paper, the analysis focuses on the last. Much has been written about the problem of PAR in OFDM systems. To summarize, in a system in which information is transmitted on a series of subcarriers near in frequency, certain degenerate data will result in all subcarriers adding coherently at a certain point in the symbol. The result is a very large peak (when N is the number of subcarriers, the peak can be N times larger than the average signal power). The problem with large PAR is that the entire system design rests on the linearity of all its components. The transmitter's power amplifier is particularly difficult to design to be linear, and typically constantly consume power proportionate to the maximum supply voltage, which also represents the limit of the input signal. Thus, PAR on the order of N implies that the power amplifier will consume DC power proportionate to N times the average output power. A particularly good summary of both the PAR problem and several mitigation techniques is found in ((4)). Typically, approaches include specialized symbol alphabets, various coding techniques (including intelligently using larger constellations), and signal recovery through non-linear equalization. Preliminary results have shown that optimal adaptation worsens the PAR problem.

The typical approach taken to combat high PAR is to simply clip the time-domain output of the IFFT. Because the maximum signals occur with low probability, it is relatively safe to clip at a certain level. Clearly, clipping is a non-linear operation that will adversely affect the system. However, for adaptive systems, no analysis has been made of what the tradeoffs are between clipping (and thus power performance) and system performance (e.g., capacity, BER, etc.). Unfortunately, it is unclear how best to determine the capacity of a clipped multiuser system. The difficulties are described in the next section, the system model.

System Model

A multiple user OFDM is assumed throughout, with perfect channel information at both transmitter and receiver (e.g., base-station and mobile), and no overhead for transmitting this information or channel allocations. Furthermore, it is assumed that there is perfect carrier and symbol synchronization, and that noise in each subcarrier is additive-white-Gaussian (AWGN). For simplicity, the same notation adopted in (2) and (3) is used, e.g., there are K users, and N subcarriers, the noise power for each user's subcarrier is assumed (wlog) to be unity, and the channel gain for user k in subcarrier n is $|h_{kn}|^2$.

For simulation purposes, further assumptions are made. First, it is assumed that the transmit power for each subcarrier is determined relative only to Rayleigh fading. Ignoring path loss and shadowing at first glance may appear negligent; however, because the rate of change of these channel characteristics is so slow relative to the fading, it may be possible to adaptively adjust the power amplifier rails (maximum and minimum values) to account for them, independent of the output of the DSP. Additionally, taking advantage of path loss and shadowing would penalize users unfairly for being in poor environments, whereas fading is a safe unifying characteristic. Second, we assume a time-correlated, frequency selective Rayleigh fading channel. The channel impulse response follows a five tap exponential decay, with each tap's phase being uniformly distributed on $(0,2\pi)$, and each tap's magnitude being an independent identically distributed Rayleigh variate. The tap magnitudes are assumed constant for one symbol period, and correlated from symbol to symbol, where correlated Rayleigh random variables are generated as in (5). Everything is scaled so that for each user the final frequency domain subcarrier channel gains have unity expectation. Finally, for peak power and PAR results, the time-domain results are oversampled by at least four times to correct for underestimation problems outlined in (4).

Sub-optimal user, bit, and power allocation are done in the frequency domain, and across both frequencies and time as in (3) (and similar to (2)). The optimal results require an impractical problem relaxation – allowing users to share subcarriers – but are shown not result in significant performance gains. If time allows, the sub-optimal user allocation scheme from (2) may be used. Several even-more-sub-optimal techniques are proposed and evaluated (e.g., equal power allocation, assigned user allocation in time (rather than all users), modified-greedy user allocation).

The problem of clipped OFDM capacity, as mentioned above, is quite difficult. The formulation of the actual optimal allocation problem will be given in the final paper. To summarize, whereas allocating (frequency-domain) power subject to a peak (time-domain) power constraint may be a convex problem, it is unclear that this is the true problem that should be solved. More likely, the correct problem is to solve the optimal allocation problem subject including a noise term that is dependent on the power allocated. This second formulation is non-convex, and thus the solution is non-obvious. For the purpose of this project, the first approach is taken, and an iterative solution is presented that should, although in very poor algorithmic time, converge to the pseudo-optimal (e.g., discrete rather than continuous) solution.

Current Results and Remaining Work

Currently, the optimal single user solutions (with the exception of clipping capacity) have been implemented. Results from this work are on the web site. These solutions form the basis for the multi-user solution (the typical algorithm involves assigning users, then letting each user allocate bits and power optimally). Remaining to be completed is the multi-user allocation algorithm implementation, and the exponential-time clipping algorithm. Though the abstract promised analytical results, these likely will have to be delayed at least till the summer.

References

1. EE360 Class Notes
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